# LETTER Special Section on Wireless Distributed Networks

# Capacity of Sectorized Distributed Networks Employing Adaptive Collaboration from Remote Antennas\*\*

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**SUMMARY** Distributed networks employing collaborative transmission (CT) from remote antennas can provide improved system capacity and cell-edge performance, by using appropriate transmission strategies. When compared to conventional non-collaborative transmission (NCT) from one base station (BS), we show that CT from two adjacent BSs can be beneficial in terms of the capacity, even when the transmission rate is normalized by the number of collaborating BSs. We further demonstrate that performing adaptive transmission (AT) between NCT and CT based on the instantaneous channel conditions provide an additional gain in capacity. The exact amount of achievable gain is quantified by the closed-form formula for the capacity distribution, which is derived using the Jacobian transformation. The presented distribution is immediately applicable to 6-sectored distributed cellular network, for which we present numerical verification of the results.

*key words:* capacity, collaborative transmission, distributed network, remote antennas, cellular system

#### 1. Introduction

Coordinated multipoint transmission and reception (CoMP) technology is actively investigated as a means to enhance the system capacity and cell-edge user equipment (UE) performance, and several types of CoMP scenarios have been proposed by the LTE-Advanced standard [1]. When the serving base station (BS) and neighboring BSs share both the data and channel state information (CSI) of a cell-edge UE, fully coordinated multi-cell transmission is possible, e.g., multicell spatial multiplexing, network precoding, and macro-diversity transmission. The closed-loop operation provides a significant performance gain in terms of the cell-edge throughput and average sector throughput at the expense of the feedback information of UEs to the collaborating BSs, while the open-loop operation reduces the implementation complexity.

In this letter, we consider an effective CoMP operation strategies over a sectorized distributed network [2] with full frequency reuse (reuse factor of 1). We first apply maximum ratio transmission [3] for signaling from two collab-

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orative antennas, which maximizes the receiver signal-tointerference-plus-noise ratio (SINR) when the CSI is available at the transmitter. We derive statistical distributions for the output SINR, the corresponding instantaneous capacity, and the ergodic capacity, all in closed-form when the collaborative transmission (CT) is employed. When the CT performance is compared with non-collaborative transmission (NCT) performance, we use the capacity normalized by the number of collaborating BSs as the performance measure, in order for the fair evaluation taking the BS resource into account.

Although the CT from two BSs provides a significant performance enhancement over a certain geographical range, we also observe that CT can be outperformed by NCT. This situation motivates the development of adaptive transmission (AT) between NCT and CT according to the capacity maximization criteria; for the UE located close to a certain BS, NCT can be a preferable mode of operation, whereas CT can provide an increased efficiency for some geographic locations. It is worthwhile to note that the geographic location of the UE is not the only criterion to determine the operational mode, but the utilization of the instantaneous channel condition provides further performance gain. To analytically evaluate the amount of gain obtained by using AT, we derive the exact capacity distribution using the Jacobian transformation method [4]. Monte-Carlo simulation confirms the accuracy of the derived results and quantifies the gains obtained by the proposed transmission strategies. We assume synchronized collaborative transmission from different BSs. Performance degradation due to asynchronous nature of collaborative transmission, as discussed in [5], is not within the scope of our discussion.

#### 2. System Model

We consider a 6-sectored CoMP model as depicted in Fig. 1, where each BS has six directional antennas covering disjoint regions. Thus the BS coverage in a given cell is split into six regions, and the collaborative transmission from two directional antennas can be performed on these regions independently. One such region is highlighted for the target UE in the figure. The total number of BSs used for simulation is N = 19, reflecting a two-tier deployment. Antennas not participating in collaborative transmission act as interferers, and an ideal uniform beam pattern over each sector is assumed.

The received signal for the UE is given by

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LETTER



Fig.1 An illustration of the 6-sectored CoMP model.

$$y = \sum_{i=1}^{N} \frac{h_i}{\sqrt{d_i^{\alpha}}} x_i + z \tag{1}$$

where  $x_i$  is the transmitted symbol from BS *i* with average power  $\mathbb{E}[|x_i|^2] = P$ ,  $h_i$  is the corresponding channel gain of the fading channel modeled by independent and identically distributed complex Gaussian with unit variance, i.e.,  $h_i \sim \mathcal{N}_C(0, 1)$ ,  $d_i$  is the distance between BS *i* and the UE,  $\alpha$ is the pathloss exponent, and *z* is the additive noise with variance  $\sigma_z^2$ . Suppose two most significant transmission sources providing the largest average received power values to the target UE are BS *m* and BS *n* as illustrated in Fig. 1. Received signal in (1) can be rewritten with significant signal components as separate terms

$$y = \frac{h_m}{\sqrt{d_m^{\alpha}}} x_m + \frac{h_n}{\sqrt{d_n^{\alpha}}} x_n + \sum_{i \neq m, n}^{t} \frac{h_i}{\sqrt{d_i^{\alpha}}} x_i + z$$
$$= Y_m + Y_n + I + z \tag{2}$$

where we defined  $Y_m = h_m x_m / \sqrt{d_m^{\alpha}}$ ,  $Y_n = h_n x_n / \sqrt{d_n^{\alpha}}$ , and  $I = \sum_{i \neq m,n} h_i x_i / \sqrt{d_i^{\alpha}}$ . The average power values of  $Y_m$ ,  $Y_n$ , and I are respectively denoted by  $\sigma_m^2$ ,  $\sigma_n^2$ , and  $\sigma_l^2$ . Then we have  $\sigma_m^2 = \mathbb{E}[|h_m|^2|x_m|^2 d_m^{-\alpha}] = Pd_m^{-\alpha}$ ,  $\sigma_n^2 = \mathbb{E}[|h_n|^2|x_n|^2 d_n^{-\alpha}] = Pd_n^{-\alpha}$ , and  $\sigma_l^2 = \mathbb{E}[\sum_{i \neq m,n} |h_i|^2 |x_i|^2 d_i^{-\alpha}] = P\sum_{i \neq m,n} d_i^{-\alpha}$ .

When BS *m* and BS *n* are performing CT to the target UE with a single receive antenna, the desired signal components in (2) can be represented as  $Y_m + Y_n = \mathbf{h}\mathbf{x}$  using the macroscopic multiple-input single-output channel vector  $\mathbf{h} = \left[ h_m / \sqrt{d_m^\alpha} h_n / \sqrt{d_n^\alpha} \right]$  and the transmit signal vector for  $\mathbf{x} = [x_m \ x_n]^T$ . The model can also be generalized to multi-antenna UEs. Superscript *T* denotes the vector transpose operator. Numerical results are based on the distance between two adjacent BSs set to 2r = 500 meters, and the pathloss exponent of 3.76. The average transmit power for each sector antenna is P = 30 dBm. As mentioned, the key performance measure is the normalized capacity, in the unit of [bps/Hz/sector].

#### 3. Transmission Capacity

#### 3.1 Non-adaptive Transmission

When the maximum ratio transmission is applied for

CT from BS *m* and BS *n*, the weighting vector  $\mathbf{w} = (\sqrt{2}/||\mathbf{h}||) \left[ h_m^* / \sqrt{d_m^{\alpha}} h_n^* / \sqrt{d_n^{\alpha}} \right]^T$  matched to channel **h** is multiplied to the common transmission symbol  $x = x_m = x_n$ . The instantaneous SINR for the CT with the corresponding received signal  $y = \mathbf{h}\mathbf{w}x + I + z$  is expressed as [3]

$$\gamma_{\rm CT} = \frac{2||\mathbf{h}||^2 P}{\sigma_I^2 + \sigma_z^2} = \frac{2(|Y_m|^2 + |Y_n|^2)}{\sigma_I^2 + \sigma_z^2}.$$
(3)

The term  $|Y_m|^2 + |Y_n|^2$  in the numerator is a weighted chisquare distributed random variable, and its probability density function (PDF) is given in [6, p. 847] as  $f_{|Y_m|^2+|Y_n|^2}(u) =$  $\{\exp(-u/\sigma_m^2) - \exp(-u/\sigma_n^2)\}/(\sigma_m^2 - \sigma_n^2)$ . The PDF of  $\gamma_{\text{CT}}$  is obtained by evaluating  $\{(\sigma_I^2 + \sigma_z^2)/2\}f_{|Y_m|^2+|Y_n|^2}(\gamma(\sigma_I^2 + \sigma_z^2)/2)$ for  $\gamma > 0$  as

$$f_{\gamma_{\rm CT}}(\gamma) = \frac{\exp\left(-\frac{\gamma}{2a}\right) - \exp\left(-\frac{\gamma}{2b}\right)}{2(a-b)},\tag{4}$$

where we defined  $a = \sigma_m^2 / (\sigma_I^2 + \sigma_z^2)$  and  $b = \sigma_n^2 / (\sigma_I^2 + \sigma_z^2)$ . The cumulative distribution function (CDF) of  $\gamma_{\rm CT}$  is obtained by the integration of  $f_{\gamma_{\rm CT}}(\gamma)$  over  $[0, \gamma]$  as

$$F_{\gamma_{\text{CT}}}(\gamma) = 1 - \frac{a}{a-b} \exp\left(-\frac{\gamma}{2a}\right) + \frac{b}{a-b} \exp\left(-\frac{\gamma}{2b}\right).$$
(5)

The normalized capacity for CT is determined using the capacity formula

$$\eta_{\rm CT} = \frac{1}{2} \log_2(1 + \gamma_{\rm CT}) = \frac{1}{2} \log_2\left(1 + \frac{2(|Y_m|^2 + |Y_n|^2)}{\sigma_I^2 + \sigma_z^2}\right) (6)$$

where the multiplication factor of 1/2 is used to reflect the normalized bandwidth efficiency of CT which consumes twice the BS radio resource compared to NCT. (See [7] for related discussion.) The CDF of  $\eta_{\rm CT}$  can be obtained from (5) by using the simple change of variable  $\gamma = 2^{2\eta} - 1$ 

$$F_{\eta_{\rm CT}}(\eta) = 1 - \frac{a}{a-b} \exp\left(-\frac{2^{2\eta}-1}{2a}\right) + \frac{b}{a-b} \exp\left(-\frac{2^{2\eta}-1}{2b}\right).$$
(7)

The ergodic capacity for CT can be derived using (4) as

$$\mathbb{E}[\eta_{\rm CT}] = \int_0^\infty \frac{1}{2} \log_2(1+\gamma) f_{\gamma_{\rm CT}}(\gamma) d\gamma$$
$$= \frac{-a \exp\left(\frac{1}{2a}\right) \operatorname{Ei}\left[-\frac{1}{2a}\right] + b \exp\left(\frac{1}{2b}\right) \operatorname{Ei}\left[-\frac{1}{2b}\right]}{2(\ln 2)(a-b)}$$
(8)

where  $\operatorname{Ei}[u] = -\int_{-u}^{\infty} e^{-v}/v \, dv$  is the exponential integral function. Note the derived formulas are expressed only in terms of *a* and *b*, which are determined from any given geographical location of the UE.

## 3.2 Adaptive Transmission (AT)

For given values of  $\sigma_m^2$ ,  $\sigma_n^2$ ,  $\sigma_I^2$ , and  $\sigma_z^2$ , the SINR of CT is always greater than that of NCT, which is easily observed

$$F_{\eta_{\max}}(\eta) = 1 - \frac{a}{b2^{\eta} + a - b} \exp\left(-\frac{2^{\eta} - 1}{a}\right) + \frac{b}{a - b} \exp\left(-\frac{2^{2\eta} - 1}{2b}\right) - \frac{ab2^{\eta}}{(a - b)(b2^{\eta} + a - b)} \exp\left(-\frac{b2^{2\eta} + (a - b)2^{\eta} + (b - 2a) + (a - b)2^{-\eta}}{2ab}\right)$$
(11)  
$$f_{\eta_{\max}}(\eta) = \frac{(\ln 2)\{b2^{2\eta} + (a - b + ab)2^{\eta}\}}{(b2^{\eta} + a - b)^{2}} \exp\left(-\frac{2^{\eta} - 1}{a}\right) - \frac{(\ln 2)2^{2\eta}}{a - b} \exp\left(-\frac{2^{2\eta} - 1}{2b}\right) + \frac{(\ln 2)\{\frac{b^{2}2^{4\eta+1}}{a - b} + (3b)2^{3\eta} + (a - b)2^{2\eta} - b(1 + 2a)2^{\eta} - (a - b)\}}{2(b2^{\eta} + a - b)^{2}} \exp\left(-\frac{b2^{2\eta} + (a - b)2^{\eta} + b - 2a + (a - b)2^{-\eta}}{2ab}\right)$$
(12)

from (3) and the SINR of NCT  $\gamma_{\text{NCT}} = |Y_m|^2 / (|Y_n|^2 + \sigma_I^2 + \sigma_z^2)$ . In terms of the bandwidth efficiency, however, CT outperforms NCT only under certain channel conditions due to the duplicated usage of the BS radio resource. Therefore, the maximum data throughput occurs when an appropriate transmission method is chosen based on instantaneous channel conditions. We define the maximum normalized capacity

$$\eta_{\text{max}} = \max \{\eta_{\text{NCT}}, \eta_{\text{CT}}\}$$

for which the normalized capacity of NCT is denoted as

$$\eta_{\rm NCT} = \log_2 \left( 1 + |Y_m|^2 / (|Y_n|^2 + \sigma_I^2 + \sigma_z^2) \right). \tag{9}$$

To determine the distribution of normalized capacity  $\eta_{\text{max}}$ , we first consider the joint PDF of two dependent variables  $\eta_{\text{NCT}}$  and  $\eta_{\text{CT}}$  which include common component variables  $|Y_m|^2$  and  $|Y_n|^2$ . By simultaneously solving the equations in (6) and (9),  $|Y_m|^2$  and  $|Y_n|^2$  are expressed in terms of  $\eta_{\text{NCT}}$  and  $\eta_{\text{CT}}$  as

and

$$|Y_n|^2 = (\sigma_I^2 + \sigma_z^2) \left( -2 + 2^{-\eta} \text{NCT} + 2^{2\eta} \text{CT}^{-\eta} \text{NCT} \right) / 2.$$

 $|Y_m|^2 = (\sigma_I^2 + \sigma_z^2) (1 - 2^{-\eta} \text{NCT} - 2^{2\eta} \text{CT}^{-\eta} \text{NCT} + 2^{2\eta} \text{CT}) / 2$ 

Using the Jacobian transformation method [4], the joint PDF has the form

$$f_{\eta_{\text{NCT}},\eta_{\text{CT}}}(\omega,\varphi) = \mathcal{J}(\omega,\varphi)f_{|Y_m|^2,|Y_n|^2}(u,v)$$

where  $\mathcal{J}(\omega, \varphi) = (\partial u/\partial \omega)(\partial v/\partial \varphi) - (\partial u/\partial \varphi)(\partial v/\partial \omega) = (\ln 2)^2 2^{2\varphi-\omega} (1-2^{2\varphi})(\sigma_I^2 + \sigma_z^2)^2/2$ . Since  $|Y_m|^2$  and  $|Y_n|^2$  are independent exponential variables, we obtain

$$f_{\eta_{\text{NCT}},\eta_{\text{CT}}}(\omega,\varphi) = \frac{(\ln 2)^2 2^{2\varphi-\omega} (1-2^{2\varphi}) (\sigma_I^2 + \sigma_z^2)^2}{2\sigma_m^2 \sigma_n^2} \exp\left(-\frac{u}{\sigma_m^2} - \frac{v}{\sigma_n^2}\right) = \frac{(\ln 2)^2 2^{2\varphi-\omega} (1-2^{2\varphi})}{2ab} \times \exp\left(\frac{(2^{\omega}-2^{2\omega})(1+2^{2\varphi})}{2a} - \frac{2^{\omega}-2^{2\omega+1}+2^{2\varphi}}{2b}\right) \quad (10)$$

for  $\omega \ge 0$  and  $\varphi \ge \ln(2^{\omega+1} - 1)/\ln 4$ . The range of variables  $\omega \ge 0$  and  $\varphi \ge \ln(2^{\omega+1} - 1)/\ln 4$  for nonzero values of  $f_{\eta_{\text{NCT}},\eta_{\text{CT}}}(\omega,\varphi)$  is determined from the transformation of the range  $u \ge 0$  and  $v \ge 0$ . The CDF of  $\eta_{\text{max}}$ , denoted by  $F_{\eta_{\text{max}}}(\eta)$ , is derived using the joint PDF in (10) as

$$\begin{aligned} F_{\eta_{\max}}(\eta) &= \Pr\{\eta_{\max} \leq \eta\} = \Pr\{\eta_{\text{NCT}} \leq \eta \text{ and } \eta_{\text{CT}} \leq \eta\} \\ &= \int_0^\eta \int_{\frac{\ln(2\omega+1-1)}{\ln 4}}^\eta f_{\eta_{\text{NCT}},\eta_{\text{CT}}}(\omega,\varphi) d\varphi d\omega. \end{aligned}$$

By performing the integration and after some algebra, a closed-form expression for the CDF is obtained, which we present in (11). The corresponding PDF  $f_{\eta_{\text{max}}}(\eta)$  is obtained from the differentiation  $f_{\eta_{\text{max}}}(\eta) = \frac{d}{d\eta}F_{\eta_{\text{max}}}(\eta)$  and the result is provided in (12).

# 4. Performance Evaluation

Performance of the AT mode is evaluated in terms of ergodic capacity and shown in Fig. 2, for the UE moving along the line connecting BS *m* and BS *n*. The distance is normalized by *r*, thus the normalized distance of 1 represents the mid-point between BS *m* and BS *n*. The ergodic capacity value for the AT, denoted by  $\mathbb{E}[\eta_{\text{max}}]$ , is determined by using the PDF of  $\eta_{\text{max}}$  given in (12). In Fig. 2(a), performance comparison of the NCT, CT, and AT modes is presented, where we set  $\sigma_z^2 = 0$  assuming a severely interference-limited environment. Performance variations under different noise power  $\sigma_z^2$  are evaluated in Fig. 2(b), for which we define border signal-to-noise ratio (SNR) by  $\Gamma = Pr^{-\alpha}/\sigma_z^2$ , which is an interference-free SNR from the reference BS *m* at the cell border ( $d_m = r$ ).  $\Gamma = 0$ , 10, and 20 dB in the figure respectively represent low to high SNR regime.

The ergodic capacity values  $\mathbb{E}[\eta_{CT}]$  and  $\mathbb{E}[\eta_{NCT}]$ are respectively computed using (8) and  $\mathbb{E}[\eta_{NCT}] = a \{-\exp(1/a) \operatorname{Ei}[-1/a] + \exp(1/b) \operatorname{Ei}[-1/b]\} / \{(a-b) \ln 2\}$ . As can be observed in Fig. 2(a), capacity values tend to decrease in general as the UE moves away from BS *m*, due to the decreasing SINR. When the UE is located near BS *m*, the capacity for NCT is much larger than that for CT, since the collaborative transmission uses the duplicated resource while the SINR improvement due to the signal power from

3536

LETTER



Fig. 2 Performance of the AT mode in terms of ergodic capacity at varying distances from the reference BS: (a) Comparison with the NCT and CT modes with  $\sigma_z^2 = 0$ , (b) performance variation for the AT mode with border SNR  $\Gamma = 0$ , 10, and 20 dB.

BS n is marginal. Near the BS m coverage boundary, however, CT outperforms NCT with a significant increase in SINR which overcompensates the additional BS resource usage. Ergodic capacity  $\mathbb{E}[\eta_{\max}]$  for AT is shown to be strictly greater than both  $\mathbb{E}[\eta_{\text{NCT}}]$  and  $\mathbb{E}[\eta_{\text{CT}}]$ , and the maximum amount of achievable gain from AT is 22%, observed at the crossing point between  $\mathbb{E}[\eta_{\text{NCT}}]$  and  $\mathbb{E}[\eta_{\text{CT}}]$  at normalized distance 0.74. The simulated curves by using faded signal components from the total 19 BSs are shown in symbol '+,' indicating a good agreement with the analytic results. Figure 2(b) shows the effects of noise power  $\sigma_z^2$  on the AT performance. It is observed that the curve with  $\Gamma = 20 \text{ dB}$ is almost matched to that with  $\Gamma = \infty$  which is the same curve for AT without the effect of noise in Fig. 2(a). As  $\Gamma$  decreases, i.e., as the effect of noise becomes more substantial, we observe a decrease in ergodic capacity as well. However, we observed general performance trend and the crossing point between NCT and CT remains the same.

Figure 3 shows the capacity CDFs for the NCT, CT, and AT modes, when UEs are uniformly distributed over the triangular shaded region in Fig. 1, reflecting the sector coverage of BS *m*. Analytic results are from the numerical integration using the CDF formulas given in (7) for CT,  $F_{\eta_{\text{NCT}}}(\eta) = 1 - \{a/(a-b+b2^{\eta})\} \exp(-(2^{\eta}-1)/a)$  for NCT derived in a similar manner, and (11) for AT. Simulation results are from random generations of the UE location following the two-dimensional uniform distribution over the



**Fig.3** Capacity distributions for the UE located uniformly over the sector coverage of the reference BS.

region of interest. Two curves for  $F_{\eta_{\rm NCT}}(\eta)$  and  $F_{\eta_{\rm CT}}(\eta)$  cross at the normalized capacity value of 1.77 bps/Hz/sector, suggesting CT is the preferred mode of operation when the bandwidth efficiency is below 1.77 bps/Hz/sector. The curve for  $F_{\eta_{\rm max}}(\eta)$  lies to the right of the curves for both NCT and CT, demonstrating an improved bandwidth efficiency. From the comparison between analytic and simulation values, the derived formulas are shown to be accurate indications of the experimental CDFs.

### 5. Conclusion

Collaborative transmission using distributed antennas can provide the capacity enhancement of the system as well as the outage reduction by increasing the SINR at coverage boundaries of transmission sources. By presenting the statistical distributions, we determined SINR and capacity characteristics of the collaborative transmission schemes. In particular, the performance of adaptive transmission is investigated and corresponding ergodic capacity distributions are analytically derived, to determine the additional amount of achievable gain.

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