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A Per-User QoS Enhancement Strategy via Downlink Cooperative Transmission Using Distributed Antennas*

Byungseok LEE†, **Ju Wook JANG**†, **Sang-Gyu PARK**††, *Nonmembers, and* **Wonjin SUNG**†a), *Member*

SUMMARY In this letter, we address a strategy to enhance the signal-to-interference plus noise ratio (SINR) of the worst-case user by using cooperative transmission from a set of geographically separated antennas. Unlike previously reported schemes which are based on either the power control of individual antennas or cooperative orthogonal transmission, the presented strategy utilizes the minimum-mean-squared error (MMSE) filter structure for beamforming, which provides increased robustness to the external interference as well as the background noise at the receiver. By iteratively updating the cooperative transmission beamforming vector and power control (PC), the balanced SINR is obtained for all users, while the transmission power from each antenna also converges to within the constrained value. It is demonstrated that proposed MMSE beamforming significantly outperforms other existing schemes in terms of the achievable minimum SINR.

key words: *beamforming, power control, SINR balancing, duality*

1. Introduction

Among various forms of multi-antenna systems, distributed multiple-input multiple-output (MIMO) systems which employ geometrically separated remote antennas (RA) have been shown to efficiently improve the system performance by utilizing cooperative transmission among RAs [1]. A joint determination of the power levels and beamforming vectors is usually desired for the performance optimization, and different forms of optimization criteria exist depending on operational goals of the system. The uplink and downlink duality is a powerful tool in obtaining solutions to joint PC and beamforming problems [2], [3].

In this letter, we focus on a cooperative transmission strategy using distributed RAs such that the minimum SINR of the users communicating with a given set of RAs is maximized. This type of SINR balancing plays an important role in providing the quality-of-service (QoS) fairness among the users. In cases when the channel experiences deep fading for an extended period of time, opportunistic transmission may lead to the QoS violation, and the short term fairness cannot be guaranteed. One of the effective solutions to overcome such situations is performing the PC and beamforming based on the SINR optimization. While the downlink SINR balancing and related problems have been extensively studied under the constraint of total power requirement [3], the results do not directly apply to systems with distributed RAs since each geographically separated antenna operates with its own power amplifier with individual power constraint. In order to meet the per-antenna power constraint, zero-forcing (ZF) cooperative beamforming followed by scaling down of transmission power has been proposed in [1]. Here we address the downlink SINR balancing issue as an optimization problem and derive the equivalent uplink dual problem based on the Lagrangian duality theory, to develop an efficient solution to the problem. In particular, we adopt an MMSE-based precoder as the cooperative beamformer, to achieve the enhanced minimum SINR among target receivers. As the result, the presented minimum SINR distribution outperform those of all previously reported schemes applied to distributed RAs.

Note that superscripts $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^\dagger$ respectively denote the transpose, Hermitian transpose, and inverse of the matrix. Symbol $\succeq$ denotes the matrix inequality defined for nonnegative definite matrices.

2. System Model

We consider a downlink multi-user distributed MIMO system illustrated in Fig. 1, where N geographically distributed RAs simultaneously transmit signals to K mobile users. Each RA is wireline connected to the control center, which manages coordinated beamforming among RAs. The transmitted signal vector $\mathbf{x} = [x_1, \ldots, x_N]^T$ from N RAs is of the form $\mathbf{x} = \sum_{k=1}^{K} \mathbf{v}_k d_k$ where d_k is the data symbol for the k-th user satisfying $E|d_k|^2 = 1$ and $\mathbf{v}_k = [v_{k1}, \ldots, v_{kN}]^T$ is the

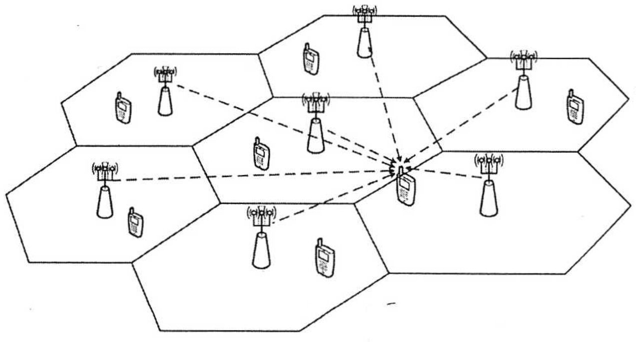

Fig. 1 Cooperative transmission from distributed remote antennas.

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†The authors are with the Department of Electronic Engineering, Sogang University, Seoul 121-742, Korea.
††The author is with the Department of Electronic Engineering, Hanyang University, Seoul 133-791, Korea.

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a) E-mail: wsung@sogang.ac.kr (Corresponding author)

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$N \times 1$ beamforming weight vector for the k-th user. When the transmission power for the n-th RA is limited within its maximum value P_n, the per-antenna power constraints are given as

$$E|x_n|^2 \leq P_n, \quad n = 1, \cdots, N. \tag{1}$$

The received signal for the k-th user can be written as

$$y_k = \mathbf{h}_k^H \mathbf{x} + z_k, \quad k = 1, \cdots, K \tag{2}$$

where $\mathbf{h}_k = [h_{k1} \cdots h_{kN}]^H$ represents the Rayleigh flat fading channel vector from RAs to the k-th user, assumed to be perfectly known at the transmitter, and z_k is the zero-mean complex Gaussian noise with variance σ^2. Signal model in (2) can also be generalized to multi-antenna receiver case. The SINR balancing problem is sated as follows:

$$\max_{\mathbf{v}_k} \quad \gamma$$

$$\text{s.t.} \quad \frac{|\mathbf{h}_k^H \mathbf{v}_k|^2}{\sum_{j \neq k} |\mathbf{h}_k^H \mathbf{v}_j|^2 + \sigma^2} \geq \gamma, \quad \forall k \tag{3}$$

$$\text{and} \quad \sum_{k=1}^{K} |v_{kn}|^2 \leq P_n, \quad \forall n \tag{4}$$

where $\mathbf{h}_k, P_n$, and σ^2 are given parameters while $\mathbf{v}_k$ and γ are design variables. Equations (3) and (4) respectively are associated with the SINR constraints and the per-antenna power constraints. The non-convexity of the downlink SINRs, which are jointly determined by the set of beamforming vectors, is one of the factors preventing straightforward solutions to the problem.

3. SINR Optimization via Lagrangian Duality

In [2], Yu and Lan provided an optimization framework for the uplink–downlink duality. With similar approach, we provide a solution to the SINR optimization problem using such notion of duality. Unlike the derivation in [2], however, we focus on the SINR optimization problem under the variable SINR constraint and the per-antenna power constraint. A mathematical expression of the dual uplink problem for the downlink SINR optimization is represented as follows.

$$\min_{\mathbf{Q}} \max_{\lambda_k} \quad \gamma$$

$$\text{s.t.} \quad \frac{\lambda_k |\mathbf{w}_k^H \mathbf{h}_k|^2}{\sum_{j \neq k}^{K} \lambda_j |\mathbf{w}_k^H \mathbf{h}_j|^2 + \mathbf{w}_k^H \mathbf{Q} \mathbf{w}_k} \geq \gamma, \forall k, \tag{5}$$

$$\sum_{k=1}^{K} \lambda_k \sigma^2 \leq \mathrm{tr}(\mathbf{\Phi}), \tag{6}$$

$$\text{and} \quad \mathrm{tr}(\mathbf{Q}\mathbf{\Phi}) \leq \mathrm{tr}(\mathbf{\Phi}) \tag{7}$$

where λ_k is the dual variable associated with the SINR constraint, and $\mathbf{w}_k$ is the dual uplink receiver beamforming vector for the k-th user. Diagonal matrix $\mathbf{\Phi} = \mathrm{diag}(P_1, \ldots, P_N)$ represents the per-antenna maximum transmit power, and $\mathbf{Q} = \mathrm{diag}(q_1, \ldots, q_N)$ is the diagonal matrix of dual variables associated with the per-antenna power constraint in the downlink problem. This dual problem can be interpreted as an uplink problem where $\lambda_k \sigma^2$ is the k-th user's transmit power and $\mathbf{Q}$ is the uncertain noise covariance matrix in the dual uplink. The equivalence between the downlink beamforming problem and its dual uplink beamforming problem can be shown by the Lagrangian duality theory in convex optimization, as given in the appendix. Based on the dual uplink problem, we develop an iterative algorithm to obtain an efficient solution. The dual problem is quasi-convex and corresponding design variables are λ_k, $\mathbf{w}_k$, $\mathbf{Q}$, and γ. A number of design variables can be reduced by the following procedure. First, optimal solution $\mathbf{w}_k$ of the dual problem under the fixed λ_k and $\mathbf{Q}$ is designed by the MMSE beamforming vector. Thus, $\mathbf{w}_k$ is represented as

$$\mathbf{w}_k = \left(\sum_{j=1}^{K} \lambda_j^{(l)} \mathbf{h}_j \mathbf{h}_j^H + \mathbf{Q}^{(l)} \right)^{\dagger} \mathbf{h}_k \tag{8}$$

where superscript l denotes the iteration index. To eliminate γ, (8) is plugged into the SINR constraint of the dual problem. With some manipulation of the SINR constraint, λ_k is given by

$$\lambda_k = \frac{1}{(1 + 1/\gamma)\mathbf{h}_k^H (\sum_{j=1}^{K} \lambda_j^{(l)} \mathbf{h}_j \mathbf{h}_j^H + \mathbf{Q}^{(l)}) \mathbf{h}_k} \tag{9}$$

and (9) is modified to

$$\lambda_k^* = \widetilde{\lambda}_k \frac{\mathrm{tr}(\mathbf{\Phi})}{\sum_{j=1}^{K} \widetilde{\lambda}_j \sigma^2} \tag{10}$$

where

$$\widetilde{\lambda}_k = \frac{1}{\mathbf{h}_k^H (\sum_{j=1}^{K} \lambda_j^{(l)} \mathbf{h}_j \mathbf{h}_j^H + \mathbf{Q}^{(l)}) \mathbf{h}_k}.$$

Since $\mathbf{w}_k$ and γ are eliminated, remaining design variables are λ_k and $\mathbf{Q}$. Equation (9) satisfies the interference function properties, and the convergence of such functions is guaranteed as shown in [3]. Therefore, λ_k is converges to a unique point.

Based on the above procedure, we provide an iterative algorithm for the dual uplink beamforming problem. We choose a sequential optimization method such that λ_k is optimized with fixed $\mathbf{Q}$ and vice versa. λ_k is found by the iterative algorithm of (10). To optimize the dual variable $\mathbf{Q}$, we need an additional mathematical manipulation. The Lagrangian dual function of $\mathbf{Q}$ is concave and its gradient is $-\mathrm{diag}(\sum_k \mathbf{v}_k \mathbf{v}_k^H - \mathbf{\Phi})$. Since the dual variable $\mathbf{Q}$ is determined by $\sum_k \mathbf{v}_k \mathbf{v}_k^H$, we need to calculate a set of downlink beamformer for the gradient. $\mathbf{v}_k$ is found by the uplink and downlink duality in terms of SINR. To utilize the duality, it is necessary to find the SINR at each iteration. The SINR γ_k of k-th user at the l-th iteration is obtained by substituting (10) into (9) as

$$\gamma_k^* = \left(\frac{1}{\lambda_k \mathbf{h}_k^H (\sum_{j=1}^{K} \lambda_j^{(l)} \mathbf{h}_j \mathbf{h}_j^H + \mathbf{Q}^{(l)}) \mathbf{h}_k - 1} \right)^{-1}. \tag{11}$$

When both the primal and dual problem obtain the optimal solution, the SINR constraints for both problems are satisfied with equality. In addition, $\mathbf{v}_k$ and $\mathbf{w}_k$ are scaled versions of each other, i.e., $\mathbf{v}_k = \sqrt{\delta_k}\mathbf{w}_k$. By substituting $\sqrt{\delta_k}\mathbf{w}_k$ into (3), K linear equations and their solutions δ_k are obtained. With the downlink beamformer $\mathbf{v}_k$, we find the gradient of $\mathbf{Q}$ and then, $\mathbf{Q}$ is updated via a sub-gradient method [4]. The proposed iterative algorithm is summarized as follows.

0) Initialize the iteration index $l = 0$, $\mathbf{Q}^{(0)}$, and $\lambda_k^{(0)}$.
1) Perform the dual uplink power control in (10).
2) Find the uplink MMSE vector in (8) based on the dual uplink power λ_k.
3) Compute the optimal SINR in (11).
4) Update the downlink beamformer as $\mathbf{v}_k = \sqrt{\delta_k}\mathbf{w}_k$.
5) Update the noise covariance matrix with step size t_l as

$$\mathbf{Q}^{(l+1)} = \left[\mathbf{Q}^{(l)} + t_l \mathrm{diag}\left\{\sum_{k=1}^{K} \mathbf{v}_k\mathbf{v}_k^H - \mathbf{\Phi}\right\}\right]_+ .$$

6) Set $l = l + 1$ and return to Step 1) until the convergence is obtained.

4. Numerical Results

Performance of the proposed algorithm is evaluated using the cellular environment with 7 remote RAs as shown in Fig. 1. A mobile user is generated using the uniform distribution within the "coverage" of each RA station depicted as a hexagon in the figure. Thus total of 7 users simultaneously receive signals from 7 RAs over the Rayleigh fading channel. Lognormal shadowing with standard deviation of 8 dB is also applied. The RA-to-RA distance is 2 km, and the pathloss exponent value of 4 is used. The maximum transmit power of each RA is 30 dBm. Several different noise levels are used for performance evaluation. The noise level is adjusted such that the user located at the coverage border of an RA (the hexagonal vertex) experiences 0, 10, and 20 dB reference signal-to-noise ratio (SNR) when no fading or shadowing effects are present. In the projected subgradient of the numerical algorithm, the step size of $1/\sqrt{l}$ is used for the l-th iteration.

Average convergence behavior is verifiable using the beamforming vector deviation $E[\sum_k \|\mathbf{v}_k^{(l)} - \mathbf{v}_k^{(l_{opt})}\|]$ representing the norm of average difference between the beamforming vector at the l-th iteration step and the optimal value, shown in Fig. 2. The curves are obtained by averaging the norm of the difference over 10,000 random generations of users and channels. The figure confirms the convergence of the proposed algorithm.

For performance comparison, we apply the PC without beamforming and ZF-based beamforming discussed in [1], as well as the simple full power (FP) transmission without PC or beamforming. The cumulative distribution functions (CDF) of the achieved minimum SINR for 4 different

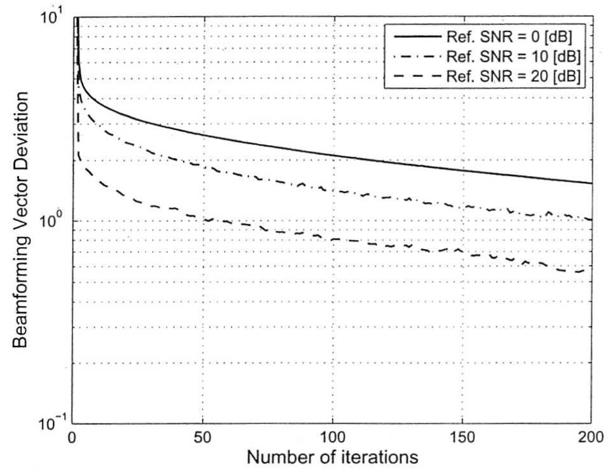

Fig. 2 Beamforming vector deviation versus the number of iterations.

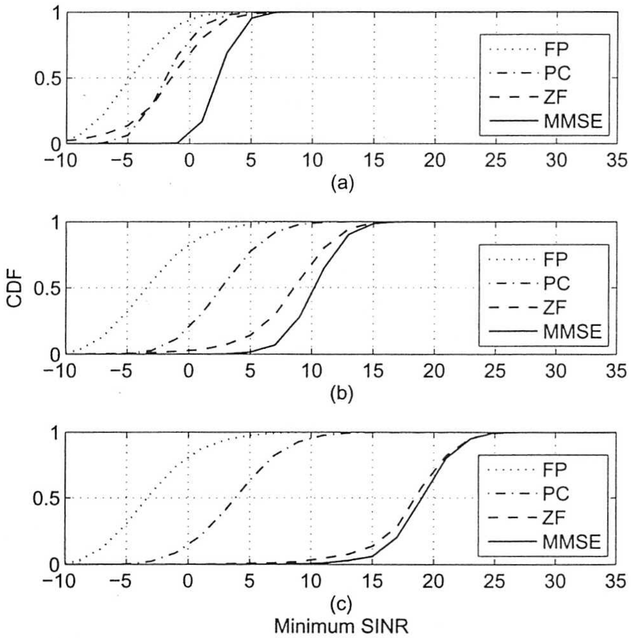

Fig. 3 Minimum SINR distributions with reference SNR of (a) 0 dB, (b) 10 dB, and (c) 20 dB.

schemes are compared in Fig. 3 using the repeated simulations. The result shows that the proposed algorithm based on multi-user MMSE beamforming with SINR balancing optimization outperforms all existing schemes. The gain is especially significant at low reference SNR values, for which the advantage of MMSE over ZF is known to be more significant.

5. Conclusion

We investigated a multi-user beamforming strategy which enhances the signal quality of all users by using geographically distributed antennas. The objective of maximizing the SINR of all users to the equivalent level is formulated as an optimization problem, and an iterative algorithm to obtain the solution is presented using the beamforming duality. The MMSE based SINR balancing algorithm is shown to outperform existing schemes including the ZF-based scheme, achieving a considerable amount of gain in SINR.

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Appendix

The primal problem is reformulated into the Lagrangian relaxation form

$$L(\mathbf{v}_k, \lambda_k, \mathbf{Q}, \gamma) = \gamma - \sum_{k=1}^{K} \lambda_k \sigma^2 + \mathrm{tr}(\mathbf{Q}\mathbf{\Phi}) + \sum_{k}^{K} \mathbf{v}_k^H \left\{ \frac{\lambda_k}{\gamma} \mathbf{h}_k \mathbf{h}_k^H - \sum_{j \neq k}^{K} \lambda_j \mathbf{h}_j \mathbf{h}_j^H - \mathbf{Q} \right\} \mathbf{v}_k \quad (A\cdot 1)$$

To investigate the zero derivative of the Karush-Kuhn-Turker (KKT) conditions, we take the gradient of L with respect to $\mathbf{v}_k$ as

$$\frac{\partial L}{\partial \mathbf{v}_k} = \left(\frac{\lambda_k}{\gamma} \mathbf{h}_k \mathbf{h}_k^H - \sum_{j \neq k}^{K} \lambda_j \mathbf{h}_j \mathbf{h}_j^H - \mathbf{Q} \right) \mathbf{v}_k = 0. \quad (A\cdot 2)$$

After some algebra, we obtain an expression for $\mathbf{v}_k$ as

$$\mathbf{v}_k = \left(\sum_{j=1}^{K} \lambda_j \mathbf{h}_j \mathbf{h}_j^H + \mathbf{Q} \right)^{\dagger} \left(1 + \frac{1}{\gamma} \right) \lambda_k \mathbf{h}_k \mathbf{h}_k^H \mathbf{v}_k. \quad (A\cdot 3)$$

We also define

$$\mathbf{w}_k = \left(\sum_{j=1}^{K} \lambda_j \mathbf{h}_j \mathbf{h}_j^H + \mathbf{Q} \right)^{\dagger} \mathbf{h}_k \quad (A\cdot 4)$$

and

$$\sqrt{\delta_k} = \left(1 + \frac{1}{\gamma} \right) \lambda_k \mathbf{h}_k^H \mathbf{v}_k. \quad (A\cdot 5)$$

By substituting $\mathbf{v}_k = \mathbf{w}_k \sqrt{\delta_k}$ into (A· 1), we have

$$L(\mathbf{v}_k, \lambda_k, \mathbf{Q}, \gamma) = \gamma - \sum_{k=1}^{K} \lambda_k \sigma^2 + \mathrm{tr}(\mathbf{Q}\mathbf{\Phi}) + \sum_{k}^{K} \delta_k \mathbf{w}_k^H \left\{ \frac{\lambda_k}{\gamma} \mathbf{h}_k \mathbf{h}_k^H - \sum_{j \neq k}^{K} \lambda_j \mathbf{h}_j \mathbf{h}_j^H - \mathbf{Q} \right\} \mathbf{w}_k. \quad (A\cdot 6)$$

When δ_k is interpreted as the Lagrangian multiplier of

$$\mathbf{w}_k^H \left\{ \frac{\lambda_k}{\gamma} \mathbf{h}_k \mathbf{h}_k^H - \sum_{j \neq k}^{K} \lambda_j \mathbf{h}_j \mathbf{h}_j^H - \mathbf{Q} \right\} \mathbf{w}_k \leq 0, \quad (A\cdot 7)$$

(A· 6) can be represented as following problem.

$$\min_{\lambda_k, \mathbf{Q}} \ \gamma - \sum_{k=1}^{K} \lambda_k \sigma^2 + \mathrm{tr}(\mathbf{Q}\mathbf{\Phi}) \quad (A\cdot 8)$$

$$\text{s.t. } \mathbf{w}_k^H \left\{ \frac{\lambda_k}{\gamma} \mathbf{h}_k \mathbf{h}_k^H - \sum_{j \neq k}^{K} \lambda_j \mathbf{h}_j \mathbf{h}_j^H - \mathbf{Q} \right\} \mathbf{w}_k \leq 0 \ \forall k, \quad (A\cdot 9)$$

$$\text{and } \ \mathbf{Q} \succeq 0.$$

Next,with another design variable μ, the above problem can be rewritten as

$$\max_{\mu} \min_{\lambda_k, \mathbf{Q}} \ \gamma + \mathrm{tr}(\mathbf{Q}\mathbf{\Phi}) - \mu \quad (A\cdot 10)$$

$$\text{s.t. } \mathbf{w}_k^H \left\{ \frac{\lambda_k}{\gamma} \mathbf{h}_k \mathbf{h}_k^H - \sum_{j \neq k}^{K} \lambda_j \mathbf{h}_j \mathbf{h}_j^H - \mathbf{Q} \right\} \mathbf{w}_k \leq 0 \ \forall k, \quad (A\cdot 11)$$

$$\sum_{k=1}^{K} \lambda_k \sigma^2 \leq \mu,$$

$$\text{and } \mathbf{Q} \succeq 0.$$

Finally, by substituting $\mu \mathrm{tr}(\mathbf{\Phi}) \leftarrow \mu$, $\mu \mathbf{Q} \leftarrow \mathbf{Q}$, and $\mu \lambda_k \leftarrow \lambda_k$, the problem is represented as

$$\max_{\mu} \min_{\lambda_k, \mathbf{Q}} \ \gamma + \mu(\mathrm{tr}(\mathbf{Q}\mathbf{\Phi}) - \mathrm{tr}(\mathbf{\Phi})) \quad (A\cdot 12)$$

$$\text{s.t. } \mathbf{w}_k^H \left\{ \frac{\lambda_k}{\gamma} \mathbf{h}_k \mathbf{h}_k^H - \sum_{j \neq k}^{K} \lambda_j \mathbf{h}_j \mathbf{h}_j^H - \mathbf{Q} \right\} \mathbf{w}_k \leq 0 \ \forall k, \quad (A\cdot 13)$$

$$\sum_{k=1}^{K} \lambda_k \sigma^2 \leq \mathrm{tr}(\mathbf{\Phi}), \quad (A\cdot 14)$$

$$\text{and } \ \mathbf{Q} \succeq 0.$$

Here, the maximization over μ is combined into the minimization over $\mathbf{Q}$ with the constraint $\mathrm{tr}(\mathbf{Q}\mathbf{\Phi}) - \mathrm{tr}(\mathbf{\Phi}) \leq 0$. Also, the reversal of the set of SINR constraints and the reversal of the minimization over λ_k with fixed $\mathbf{Q}$ do not affect the optimal solution. Therefore, the dual problem is derived.