A Per-User QoS Enhancement Strategy via Downlink Cooperative Transmission Using Distributed Antennas*

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1. Introduction

Among various forms of multi-antenna systems, distributed multiple-input multiple-output (MIMO) systems which employ geometrically separate remote antennas (RA) have been shown to efficiently improve the system performance by utilizing cooperative transmission among RAs [1]. A joint determination of the power levels and beamforming vectors is usually desired for the performance optimization, and different forms of optimization criteria exist depending on operational goals of the system. The uplink and downlink duality is a powerful tool in obtaining solutions to joint PC and beamforming problems [2], [3].

In this letter, we focus on a cooperative transmission strategy using distributed RAs such that the minimum SINR of the users communicating with a given set of RAs is maximized. This type of SINR balancing plays an important role in providing the quality-of-service (QoS) fairness among the users. In cases when the channel experiences deep fading for an extended period of time, opportunistic transmission may lead to the QoS violation, and the short term fairness cannot be guaranteed. One of the effective solutions to overcome such situations is performing the PC and beamforming based on the SINR optimization. While the downlink SINR balancing and related problems have been extensively studied under the constraint of total power requirement [3], the results do not directly apply to systems with distributed RAs since each geographically separated antenna operates with its own power amplifier with individual power constraint. In order to meet the per-antenna power constraint, zero-forcing (ZF) cooperative beamforming followed by scaling down of transmission power has been proposed in [1]. Here we address the downlink SINR balancing issue as an optimization problem and derive the equivalent uplink dual problem based on the Lagrangian duality theory, to develop an efficient solution to the problem. In particular, we adopt an MMSE-based precoder as the cooperative beamformer, to achieve the enhanced minimum SINR among target receivers. As the result, the presented minimum SINR distribution outperform those of all previously reported schemes applied to distributed RAs.

Note that superscripts (·)T, (·)H, and (·)† respectively denote the transpose, Hermitian transpose, and inverse of the matrix. Symbol ≥ denotes the matrix inequality defined for nonnegative definite matrices.

2. System Model

We consider a downlink multi-user distributed MIMO system illustrated in Fig. 1, where $N$ geographically distributed RAs simultaneously transmit signals to $K$ mobile users. Each RA is wireline connected to the control center, which manages coordinated beamforming among RAs. The transmitted signal vector $\mathbf{x} = [x_1, \ldots, x_N]^T$ from $N$ RAs is of the form $\mathbf{x} = \sum_{k=1}^{K} \mathbf{v}_k d_k$ where $d_k$ is the data symbol for the $k$-th user satisfying $E|d_k|^2 = 1$ and $\mathbf{v}_k = [v_{k1}, \ldots, v_{KN}]^T$ is the transmitted signal vector from a set of geographically separated antennas.

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N × 1 beamforming weight vector for the k-th user. When
the transmission power for the n-th RA is limited within its
maximum value \( P_n \), the per-antenna power constraints are
given as
\[ E |x_k|^2 \leq P_n, \quad n = 1, \ldots, N. \tag{1} \]
The received signal for the k-th user can be written as
\[ y_k = h_k^H x + z_k, \quad k = 1, \ldots, K \tag{2} \]
where \( h_k = [h_{k1}, \ldots, h_{kN}]^H \) represents the Rayleigh flat fading
channel vector from RAs to the k-th user, assumed to be
perfectly known at the transmitter, and \( z_k \) is the zero-mean complex
Gaussian noise with variance \( \sigma^2 \). Signal model in (2) can also be
generalized to multi-antenna receiver case. The SINR balancing
problem is stated as follows:
\[
\begin{align*}
\text{max}_{\gamma} & \quad \gamma \\
\text{s.t.} & \quad \frac{|h_k^H v_k|^2}{\sum_{k \neq k'} |h_k^H v_{k'}|^2 + \sigma^2} \geq \gamma, \quad \forall k, \tag{3} \\
\text{and} & \quad \sum_{k=1}^K |v_k|^2 \leq P_n, \quad \forall n \tag{4}
\end{align*}
\]
where \( h_k, P_n, \) and \( \sigma^2 \) are given parameters while \( v_k \) and
\( \gamma \) are design variables. Equations (3) and (4) respectively
are associated with the SINR constraints and the per-
antenna power constraints. The non-convexity of the down-
link SINRs, which are jointly determined by the set of beam-
forming vectors, is one of the factors preventing straightforward solutions to the problem.

3. SINR Optimization via Lagrangian Duality

In [2], Yu and Lan provided an optimization framework for
the uplink–downlink duality. With similar approach, we
provide a solution to the SINR optimization problem using
such notion of duality. Unlike the derivation in [2], however,
we focus on the SINR optimization problem under the vari-
able SINR constraint and the per-antenna power constraint.
A mathematical expression of the dual uplink problem for
the downlink SINR optimization is represented as follows.
\[
\begin{align*}
\min_{Q} & \quad \max_{\lambda_k} \gamma \\
\text{s.t.} & \quad \lambda_k |w_k^H h_k|^2 \geq \gamma, \quad \forall k, \tag{5} \\
& \quad \sum_{k=1}^K \lambda_k |w_k^H h_k|^2 + w_k^H Q w_k = \gamma^2, \quad \forall k, \tag{6} \\
& \quad \lambda_k \sigma^2 \leq \text{tr}(\Phi), \tag{7} \\
& \quad \text{and} \quad \text{tr}(Q \Phi) \leq \text{tr}(\Phi)
\end{align*}
\]
where \( \lambda_k \) is the dual variable associated with the SINR
constraint, and \( w_k \) is the dual uplink receiver beamforming vec-
tor for the k-th user. Diagonal matrix \( \Phi = \text{diag}(P_1, \ldots, P_N) \)
represents the per-antenna maximum transmit power, and
\( Q = \text{diag}(q_1, \ldots, q_N) \) is the diagonal matrix of dual vari-
ables associated with the per-antenna power constraint in
the downlink problem. This dual problem can be interpreted
as an uplink problem where \( \lambda_k \sigma^2 \) is the k-th user’s transmit
power and \( Q \) is the uncertain noise covariance matrix in the
dual uplink. The equivalence between the downlink beam-
forming problem and its dual uplink beamforming problem
can be shown by the Lagrangian duality theory in convex
optimization, as given in the appendix. Based on the dual
uplink problem, we develop an iterative algorithm to obtain
an efficient solution. The dual problem is quasi-convex and
the corresponding design variables are \( \lambda_k, w_k, Q, \) and \( \gamma \). A num-
der of design variables can be reduced by the following pro-
ficiency. First, optimal solution \( w_k \) of the dual problem under
the fixed \( \lambda_k \) and \( Q \) is designed by the MMSE beamforming vector. Thus, \( w_k \) is represented as
\[
w_k = \left( \sum_{j=1}^K \lambda_j^0 h_j h_j^H + Q^{(0)} \right)^{-1} h_k \tag{8}
\]
where superscript \( l \) denotes the iteration index. To eliminate
\( \gamma \), (8) is plugged into the SINR constraint of the dual problem.
With some manipulation of the SINR constraint, \( \lambda_k \) is given by
\[
\lambda_k = \frac{1}{(1 + 1/\gamma) h_k^H (\sum_{j=1}^K \lambda_j^0 h_j h_j^H + Q^{(0)}) h_k}
\tag{9}
\]
and (9) is modified to
\[
\lambda_k^* = \lambda_k \frac{\text{tr}(\Phi)}{\sum_{j=1}^K \lambda_j^0 \sigma^2} \tag{10}
\]
where
\[
l_k = \frac{1}{h_k^H (\sum_{j=1}^K \lambda_j^0 h_j h_j^H + Q^{(0)}) h_k}
\]
Since \( w_k \) and \( \gamma \) are eliminated, remaining design variables
are \( \lambda_k \) and \( Q \). Equation (9) satisfies the interference function
properties, and the convergence of such functions is guaran-
teed as shown in [3]. Therefore, \( \lambda_k \) is converges to a unique
point.

Based on the above procedure, we provide an iterative
algorithm for the dual uplink beamforming problem. We
choose a sequential optimization method such that \( \lambda_k \) is opti-
mized with fixed \( Q \) and vice versa. \( \lambda_k \) is found by the
iterative algorithm of (10). To optimize the dual variable
\( Q \), we need an additional mathematical manipulation. The
Lagrangian dual function of \( Q \) is concave and its gradient
is \( -\text{diag}(\sum_k v_k v_k^H - \Phi) \). Since the dual variable \( Q \) is deter-
mined by \( \sum_k v_k v_k^H \), we need to calculate a set of downlink
beamformer for the gradient. \( v_k \) is found by the uplink and
downlink duality in terms of SINR. To utilize the duality, it
is necessary to find the SINR at each iteration. The SINR
\( \gamma_k \) of k-th user at the l-th iteration is obtained by substituting
(10) into (9) as
\[
\gamma_k^* = \left( \frac{1}{\bar{\lambda}_k h_k^H (\sum_{j=1}^K \lambda_j^0 h_j h_j^H + Q^{(0)}) h_k - 1} \right)^{-1}
\tag{11}
\]
When both the primal and dual problem obtain the optimal solution, the SINR constraints for both problems are satisfied with equality. In addition, \( v_k \) and \( w_k \) are scaled versions of each other, i.e., \( v_k = \sqrt{\delta_k} w_k \). By substituting \( \sqrt{\delta_k} w_k \) into (3), \( K \) linear equations and their solutions \( \delta_k \) are obtained. With the downlink beamformer \( v_k \), we find the gradient of \( Q \) and then, \( Q \) is updated via a sub-gradient method [4]. The proposed iterative algorithm is summarized as follows.

0) Initialize the iteration index \( l = 0 \), \( Q^{(0)} \), and \( \delta_k^{(0)} \).
1) Perform the dual uplink power control in (10).
2) Find the uplink MMSE vector in (8) based on the dual uplink power \( \lambda_k \).
3) Compute the optimal SINR in (11).
4) Update the downlink beamformer as \( v_k = \sqrt{\delta_k} w_k \).
5) Update the noise covariance matrix with step size \( t_l \) as

\[
Q^{(l+1)} = \left[ Q^{(l)} + t_l \mathcal{D} \left( \sum_{k=1}^{K} v_k v_k^H - \Phi \right) \right]_{+}
\]

6) Set \( l = l + 1 \) and return to Step 1) until the convergence is obtained.

### 4. Numerical Results

Performance of the proposed algorithm is evaluated using the cellular environment with 7 remote RAs as shown in Fig. 1. A mobile user is generated using the uniform distribution within the "coverage" of each RA station depicted as a hexagon in the figure. Thus total of 7 users simultaneously receive signals from 7 RAs over the Rayleigh fading channel. Lognormal shadowing with standard deviation of 8 dB is also applied. The RA-to-RA distance is 2 km, and the pathloss exponent value of 4 is used. The maximum transmit power of each RA is 30 dBm. Several different noise levels are used for performance evaluation. The noise level is adjusted such that the user located at the coverage border of an RA (the hexagonal vertex) experiences 0, 10, and 20 dB reference signal-to-noise ratio (SNR) when no fading or shadowing effects are present. In the projected subgradient of the numerical algorithm, the step size of \( 1/\sqrt{l} \) is used for the \( l \)-th iteration.

Average convergence behavior is verifiable using the beamforming vector deviation \( E[\sum_k ||v_k^{(l)} - v_k^{(l-1)}||^2] \) representing the norm of average difference between the beamforming vector at the \( l \)-th iteration step and the optimal value, shown in Fig. 2. The curves are obtained by averaging the norm of the difference over 10,000 random generations of users and channels. The figure confirms the convergence of the proposed algorithm.

For performance comparison, we apply the PC without beamforming and ZF-based beamforming discussed in [1], as well as the simple full power (FP) transmission without PC or beamforming. The cumulative distribution functions (CDF) of the achieved minimum SINR for 4 different schemes are compared in Fig. 3 using the repeated simulations. The result shows that the proposed algorithm based on multi-user MMSE beamforming with SINR balancing optimization outperforms all existing schemes. The gain is especially significant at low reference SNR values, for which the advantage of MMSE over ZF is known to be more significant.

### 5. Conclusion

We investigated a multi-user beamforming strategy which enhances the signal quality of all users by using geographically distributed antennas. The objective of maximizing the SINR of all users to the equivalent level is formulated as an optimization problem, and an iterative algorithm to obtain
the solution is presented using the beamforming duality. The MMSE based SINR balancing algorithm is shown to outperform existing schemes including the ZF-based scheme, achieving a considerable amount of gain in SINR.

References


Appendix

The primal problem is reformulated into the Lagrangian relaxation form

\[ L(v_k, \lambda_k, Q, \gamma) = \gamma - \sum_{k=1}^{K} \lambda_k \sigma^2 + \text{tr}(Q \Phi) \]
\[ + \sum_{k} v_k^H \left( \frac{\lambda_k}{\gamma} h_k h_k^H - \sum_{j \neq k} \lambda_j h_j h_j^H - Q \right) v_k \]  \hspace{1cm} (A·1)

To investigate the zero derivative of the Karush-Kuhn-Tucker (KKT) conditions, we take the gradient of \( L \) with respect to \( v_k \) as

\[ \frac{\partial L}{\partial v_k} = \left( \frac{\lambda_k}{\gamma} h_k h_k^H - \sum_{j \neq k} \lambda_j h_j h_j^H - Q \right) v_k = 0. \]  \hspace{1cm} (A·2)

After some algebra, we obtain an expression for \( v_k \) as

\[ v_k = \left( \sum_{j=1}^{K} \lambda_j h_j h_j^H + Q \right) \left( 1 + \frac{1}{\gamma} \right) \lambda_k h_k h_k^H v_k. \]  \hspace{1cm} (A·3)

We also define

\[ w_k = \left( \sum_{j=1}^{K} \lambda_j h_j h_j^H + Q \right) h_k \]  \hspace{1cm} (A·4)

and

\[ \sqrt{\delta_k} = \left( 1 + \frac{1}{\gamma} \right) \lambda_k h_k h_k^H v_k. \]  \hspace{1cm} (A·5)

By substituting \( v_k = w_k \sqrt{\delta_k} \) into (A·1), we have

\[ L(v_k, \lambda_k, Q, \gamma) = \gamma - \sum_{k=1}^{K} \lambda_k \sigma^2 + \text{tr}(Q \Phi) \]
\[ + \sum_{k} \delta_k w_k^H \left( \frac{\lambda_k}{\gamma} h_k h_k^H - \sum_{j \neq k} \lambda_j h_j h_j^H - Q \right) w_k. \]  \hspace{1cm} (A·6)

When \( \delta_k \) is interpreted as the Lagrangian multiplier of

\[ w_k^H \left( \frac{\lambda_k}{\gamma} h_k h_k^H - \sum_{j \neq k} \lambda_j h_j h_j^H - Q \right) w_k \leq 0, \]  \hspace{1cm} (A·7)

(A·6) can be represented as following problem.

\[ \min_{\lambda_k, Q} \gamma - \sum_{k=1}^{K} \lambda_k \sigma^2 + \text{tr}(Q \Phi) \]
\[ \text{s.t. } w_k^H \left( \frac{\lambda_k}{\gamma} h_k h_k^H - \sum_{j \neq k} \lambda_j h_j h_j^H - Q \right) w_k \leq 0 \forall k, \]  \hspace{1cm} (A·9)

and \( Q \succeq 0 \).

Next, with another design variable \( \mu \), the above problem can be rewritten as

\[ \max_{\mu} \min_{\lambda_k, Q} \gamma + \text{tr}(Q \Phi) - \mu \]
\[ \text{s.t. } w_k^H \left( \frac{\lambda_k}{\gamma} h_k h_k^H - \sum_{j \neq k} \lambda_j h_j h_j^H - Q \right) w_k \leq 0 \forall k, \]  \hspace{1cm} (A·11)

Finally, by substituting \( \mu \text{tr}(\Phi) \leftarrow \mu, M \Phi \leftarrow Q, \) and \( \mu \lambda_k \leftarrow \lambda_k, \) the problem is represented as

\[ \max_{\mu} \min_{\lambda_k, Q} \gamma + \mu \text{tr}(Q \Phi) - \text{tr}(\Phi) \]
\[ \text{s.t. } w_k^H \left( \frac{\lambda_k}{\gamma} h_k h_k^H - \sum_{j \neq k} \lambda_j h_j h_j^H - Q \right) w_k \leq 0 \forall k, \]  \hspace{1cm} (A·13)

and \( Q \succeq 0 \).

Here, the maximization over \( \mu \) is combined into the minimization over \( Q \) with the constraint \( \text{tr}(Q \Phi) - \text{tr}(\Phi) \leq 0 \). Also, the reversal of the set of SINR constraints and the reversal of the minimization over \( \lambda_k \) fixed \( Q \) do not affect the optimal solution. Therefore, the dual problem is derived.